Calculation of Blade-Vortex Interaction Airloads on Helicopter Rotors

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Two alternative approaches are developed to calculate blade-vortex interaction airloads on helicopter rotors: second-order lifting-line theory, and a lifting-surface theory correction. The common approach of using a larger vortex core radius to account for lifting-surface effects is quantified. The second-order lifting-line theory also improves the modeling of low aspect-ratio blades, yawed flow, and swept tips. Calculated results are compared with wind-tunnel measurements of lateral flapping, and with flight test measurements of blade section lift on SA349/2 and H-34 helicopter rotors. The tip vortex core radius required for good correlation with the flight test data is about 20% chord, which is within the range of measured viscous core sizes for helicopter rotors.

Nomenclature

C_T	= rotor thrust coefficient, thrust/$[\rho(\Omega R)^2\pi R^2]$
c	= rotor blade chord
$d(C_T/s)/dr$	= blade section lift, $L/[\rho(\Omega R)^2 c]$
L	= blade section lift (force per unit span)
M_{at}	= advancing tip Mach number, $(V+\Omega R)/$(sound speed)
N	= number of blades
R	= blade radius
V	= flight speed
μ	= rotor advance ratio, $V/(\Omega R)$
ρ	= air density, or blade radial station
σ	= rotor solidity, $Nc/\pi R$
ψ	= rotor azimuth angle, measured from downstream in direction of rotor rotation
Ω	= rotor rotational speed

Introduction

THE vortex wake of the rotor is a factor in most problems of helicopters, including blade loads, vibration, noise, and even performance. The wake, particularly the discrete tip vortices, is a principal source of the higher harmonic loading on the blades. Calculating the wake-induced velocities at the rotor using a realistic representation of the vorticity was one of the first applications of digital computers to helicopter aerodynamics,[1,2] and has been a continuing subject for research. A key aspect of such calculations, and the subject of the present paper, is the blade airloads produced by the tip vortices in the rotor wake.

The aerodynamics and wake models of a modern comprehensive helicopter analysis[3] have the following features. The rotor aerodynamic model is based on lifting-line theory, using steady two-dimensional airfoil characteristics and a vortex wake. The model includes an empirical dynamic stall model, a yawed flow correction, and unsteady aerodynamic forces from thin-airfoil theory. The rotor wake model is based on a vortex lattice (straight-line segment) approximation for the wake. A small viscous core radius is used for the tip vortices. A large core size is used for the inboard wake elements, not as a representation of a physical effect, but to produce an approximation for sheet elements. Sheet elements can be used, but are not usually necessary. A model of the wake rollup process is included. Eventually the tip vortex has strength equal to the maximum bound circulation of the azimuth where the wake element was trailed. A number of parameters allow the tip vortex to have only a fraction of this maximum strength when it encounters the following blade, with the remainder of the vorticity still in the inboard wake. The radial location of the tip vortex formation at the generating blade is also prescribed in the model. The wake geometry models include simple undistorted geometry and calculated free wake. The free-wake analysis calculates the distorted tip vortex geometry for a single rotor in forward flight.

Many factors determine the magnitude of the vortex-induced loading on a rotor blade, including the extent of the tip vortex rollup, the tip vortex strength, the viscous core size, lifting-surface theory effects on the induced loading, and possibly even vortex bursting or vortex-induced stall on the blade. The tip vortex core size determines the maximum tangential velocity of the vortex, but it is usually an input parameter of the analysis, since it is seldom either measured or calculated. Hence, the core size is a convenient parameter to be used in the analysis to control the amplitude of the calculated blade-vortex interaction loads. Then, the core size represents not only the actual viscous core radius but also all phenomena of the interaction that are not otherwise modeled. With this approach, correlation between measured and calculated rotor behavior is best when a core size is used that is clearly much larger than the actual viscous core, typically 70% chord.[4,5] Such an empirical approach has limitations that are unacceptable in the long term, such as not being able to select the correct core size until after the loads have been measured, and not being able to account for other effects such as swept or yawed flow. The long-term goal is to improve the wing and wake models to the point where the vortex core in the analysis simply represents the actual physical core and nothing else.

The present investigation introduces two alternative methods for improving the wing model for blade-vortex interaction: second-order lifting-line theory, and a lifting-surface theory correction. The intent is to improve the calculation of the airloads without actually resorting to methods such as lifting-surface theory, which require more computation and still need some development for rotary wing applications. Calculated results are compared with wind-tunnel measurements of lateral flapping, and with flight test measurements of blade section lift on SA349/2 and H-34 helicopter rotors.

Second-Order Lifting-Line Theory

Several investigations (summarized in Ref. 6) have shown that second-order lifting-line theory gives nearly the same results as lifting-surface theory, including the lift produced in close blade-vortex interactions.[7] In addition, second order lifting-line theory should improve the loads calculated for swept tips, yawed flow, and low aspect-ratio blades. Formal

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lifting-line theory is the solution of the three-dimensional wing loading problem using the method of matched asymptotic expansions. Based on the assumption of large wing aspect ratio, the problem is split into separate outer (wake) and inner (wing) problems, which are solved individually and then combined through a matching procedure. For a rotor in forward flight it is also necessary to consider a swept and yawed planform, and unsteady, compressible, and viscous flow. The lowest-order fixed wing solution is Prandtl's theory (steady and no sweep). The development of higher-order lifting-line theories originated with Weissinger for intuitive methods, and with Van Dyke for singular perturbation methods.

The lifting-line theories found in the literature, although including unsteady, transonic, and swept flow, are generally analytical methods, i.e., they obtain analytical solutions for both the inner and outer problems, and are in quadrature rather than integral form. For the rotary wing, however, it is necessary to include stall (high angle of attack) in the inner solution, and the helical, distorted, rolled-up wake geometry in the outer solution. Hence, for the rotor problem, the objective is to obtain from lifting-line theory a separate formulation of the inner and outer problems, with numerical, not analytical, solutions, and a matching procedure that will be the basis for an iterative solution.

Lifting-line theory contains so many approximations and assumptions that, ultimately, it must be replaced by a more accurate method, but that is in the future. Presently, lifting-line theory is still the only practical method for including viscous effects in the analysis, allowing two-dimensional experimental data containing information on stall and compressibility to be combined with three-dimensional wakes. And so it is important to develop the best possible lifting-line theory for the rotor. Higher-order perturbation methods are not directly applicable, since they involve analytical solutions and quadrature rather than integral form. The objective is to define a practical method, one that gives the best accuracy without convergence problems or singularities. For this purpose, the perturbation solutions provide a guide and a sound mathematical foundation for the approach.

The development and results of higher-order lifting-line theory are summarized in Ref. 6, including citations of the sources responsible for developing various aspects of the theory. Based on Ref. 6, the following is a practical implementation of second-order lifting-line theory for rotors.

The outer problem is an incompressible vortex wake behind a lifting-line, with distorted geometry and rollup. The lifting-line (bound vortex) is at the quarter chord, as an approximation for the quadrupole line introduced by second-order loading. The trailed wake begins at the bound vortex, whereas the shed wake is created a quarter chord aft of the collocation point on the wing (the lifting-line approximation for unsteady loading). Three components of wake-induced velocity are evaluated at the collocation points, excluding the contribution of the bound vortex. The collocation points are at the three-quarter chord (in the direction of the local flow) as an approximation for a linearly varying induced velocity introduced by the second-order wake.

The inner problem consists of unsteady, compressible, viscous flow about an infinite aspect-ratio wing, and in a uniform flow consisting of the yawed freestream and three components of induced velocity. This problem is split into several parts: two-dimensional, steady, compressible, viscous flow (airfoil tables); and corrections for unsteady flow (small-angle noncirculatory loads, but without any shed wake), dynamic stall, and yawed flow (equivalence assumption for a swept wing).

This formulation is generally second-order accurate for lift, including the effects of sweep and yaw, but less accurate for section moment. Note that placing the collocation points at the quarter chord gives a first-order lifting-line theory.

Implementation of this lifting-line theory primarily involves the wake model directly behind the blade at which the induced velocity is being calculated. A vortex lattice is used for this part of the wake, rather than wake sheet elements. The discretization of the wake is not well controlled using sheet elements, because of edge and corner singularities, particularly when planar-rectangular elements are used. Also, the most important case of the downwash from the two sheet panels adjoining the collocation point would require a higher-order element or some other special treatment. By the time the wake reaches a following blade, however, the model changes to one representing the rolled-up vorticity: the tip vortices are discrete line segments with a finite core radius, and if necessary sheet elements can be used for the inboard wake. Details of the implementation are given in Ref. 8.

Lifting-Surface Theory Correction

The second method considered to improve the wing model for blade-vortex interactions is a lifting-surface theory correction based on the theory of Ref. 9. Note that the lifting-surface theory correction is an alternative to the second-order lifting-line theory developed in the preceding section; the two methods should not be used simultaneously.

The basis for the lifting-surface theory correction is a model problem consisting of an infinite wing encountering a straight infinite vortex with intersection angle Λ. The wing and vortex lie in parallel planes with separation h. Reference 9 derived a lifting-surface theory solution for this model problem and developed an approximation suitable for vortex-induced loads. By applying a Fourier transform to the vortex-induced downwash and lift, the problem becomes finding the loading produced by a sinusoidal gust. A numerical solution to this problem was approximated by a series that has an analytical inverse transform. This produced an approximate lifting-surface solution L_{ls} for vortex-induced loads. A similar expression is obtained for the vortex-induced load that would be obtained from lifting-line theory L_{ll}. Then the induced velocity of each line segment of the tip vortex in the wake model is multiplied by the factor L_{ls}/L_{ll}. By this means, the lifting-line calculation of the vortex-induced loads should give the lifting-surface theory result.

The vortex-induced downwash in the wing plane is proportional to $r/(r^2 + h_e^2)$, where r is the distance from the vortex line, and h_e is an effective vertical separation (including the influence of the viscous core). The Fourier transform (over r) of this downwash distribution is then proportional to exp $(-\nu h)$, where ν is the wave number ($2\pi/\nu$ is the wavelength of the sinusoidal gust; r, h_e, and ν are scaled with the wing semichord). Hence, wave numbers in the range $\nu = 0$–5 are considered relevant for vortex-wing separations greater than a quarter chord. The lifting-surface solution is obtained in terms of the function $g_L(\nu)$, which is the ratio of the wing section lift to the two-dimensional, quasistatic lift that would be produced by the sinusoidal gust. For the parallel intersection (two-dimensional, unsteady airfoil), the wave number becomes the reduced frequency, and g_L is the Sears function.

The generalized Sears function g_L is approximated in Ref. 9 by a sum of exponentials:

$$g_L \cong \exp(-c_0\nu) - i\sin(b_0\nu)\, a_0' \exp(-c_0'\nu) + \exp[i(b_1\nu - b_2)]\,[-a_1\nu^2\exp(-c_1\nu) + a_2\nu^4\exp(-c_2\nu)]$$

This form is chosen because it (and its product with the vortex spectrum) has an analytical inverse. The exponentials decay faster than the true g_L, and so any finite series of this form will be too small at high ν. The approximation is acceptable, however, over the required range of ν [i.e., because g_L is multiplied by exp $(-\nu h)$ from the vortex spectrum]. Linear lifting-surface theory was solved numerically for g_L, and the results used to identify the coefficients in the expression as a function of intersection angle Λ and Mach number (only the incompressible results are used here).

To formulate a lifting-surface correction, the corresponding lifting-line solution for g_L must be developed. For the perpendicular intersection (an infinite wing with steady, sinusoidally varying downwash), Prandtl's equation is readily solved for g_L, even with a discretized trailed wake. For the parallel intersection (a two-dimensional airfoil in a sinusoidal gust), it is necessary to consider the lifting-line approximation for unsteady aerodynamics.[10] With the downwash evaluated at a single point on the airfoil, an approximation for the Sears function (g_L here) is obtained that is good at least to a reduced frequency of one-half.

The lifting-line theory expression for g_L is based directly on the lifting-surface theory expression (details are given in Ref. 8). The approach was to adjust the constants from the lifting-line expression in order to match the lifting line g_L for the perpendicular interaction (the discretized wake result) and the parallel interaction (up to reduced frequencies of about 1). The low-frequency limit in both cases gives $c_0 = \pi/2$. The factors a_1 and a_2 were increased in order to match the magnitude of g_L for wave numbers around 1. The factor a_0' was decreased for the parallel interaction in order to match the peak phase shift at very low frequency. Finally, for the parallel interaction, $b_2 = 0$ and $b_1 = 0.5$ are used to match the phase of g_L for wave numbers around 1. Hence, the expression for the lifting-surface theory lift L_{ls} is used with

$$c_0 = 1.571$$

$$a_1 = (1.25 + 0.5\,\sin\Lambda)(a_1)_{ls}$$

$$a_2 = (2.5 + \sin\Lambda)(a_2)_{ls}$$

$$a_0' = 0.75(a_0')_{ls}$$

$$b_2 = 0$$

$$b_1 = 0.5(b_1)_{ls}$$

with the constants for the lifting-surface approximation given in Ref. 9. The result is an expression for L_{ll}. The correction factor L_{ls}/L_{ll} ranges from 0.6 to 0.8 for various interaction angles at $h = 0.25$ chord.[8]

Lateral Flapping Wind-Tunnel Test

Lateral flapping in low-speed forward flight is a sensitive measure of the effects of the rotor wake. The lateral tip-path-plane tilt of an articulated rotor depends primarily on the longitudinal gradient of the induced velocity distribution over the disk. The induced velocity in forward flight is larger at the rear of the disk than at the front, which produces larger loads at the front, hence an aerodynamic pitch moment on the rotor. An articulated rotor responds to this moment like a gyro, and so the tip-path plane tilts laterally toward the advancing side. In forward flight there is also a small lateral flapping contribution proportional to the coning angle.

The lateral flapping is underpredicted when uniform inflow is used, and even when nonuniform inflow based on undistorted wake geometry is used. Below an advance ratio of about 0.16, it is necessary to include the free wake calculation in order to obtain a good estimate of the lateral flapping.[4] There is significant self-induced distortion of the tip vortices, resulting in numerous blade-vortex interactions in which the vertical separation is a fraction of the blade chord. The result of such distortion is a much larger longitudinal gradient of the induced velocity, which produces the observed lateral flapping. In the cases considered here, the free wake geometry places the tip vortices so close to the blades that the calculated flapping is sensitive to the value of the tip vortex core radius, which determines the maximum velocity induced by the vortex. The present investigation improves the calculation of the vortex-induced loading, using methods equivalent to lifting-surface theory. Hence, it is expected that a smaller core size, closer to the physical viscous core radius, will be needed for good correlation with the improved model.

Data on the flapping of a model rotor at low speed, measured in a wind tunnel, are given in Ref. 11. The model rotor parameters and operating conditions are summarized in Table 1. Figures 1–3 compare the measured and calculated lateral flapping for three models of the blade-vortex interaction loads:

1) First-order lifting-line theory and no lifting-surface correction. This model will require a larger core size;

2) Second-order lifting-line theory and no lifting-surface correction; and

3) First-order lifting-line theory with the lifting-surface theory correction.

For each case, the tip vortex core size is varied. For case 1, a core size of about 0.05 R (70% chord) appears best, but at low-advance ratio the lateral flapping magnitude is overpredicted, and the longitudinal underpredicted, regardless of the core size. With either the second-order lifting-line theory or the lifting-surface correction, a core size of about 0.035R

Table 1 Rotor parameters and operating conditions

	Model	SA349/2	H-34
Number of blades	4	3	4
Radius, ft	2.73	17.22	28.00
Solidity ratio	0.089	0.064	0.062
Airfoil	V23010-1.58	OA209	NACA0012
C_T/σ	0.080	0.065	0.082
μ	0–0.24	0.14	0.18
M_{at}	0.40–0.50	0.72	0.70
Reference	11	12	13

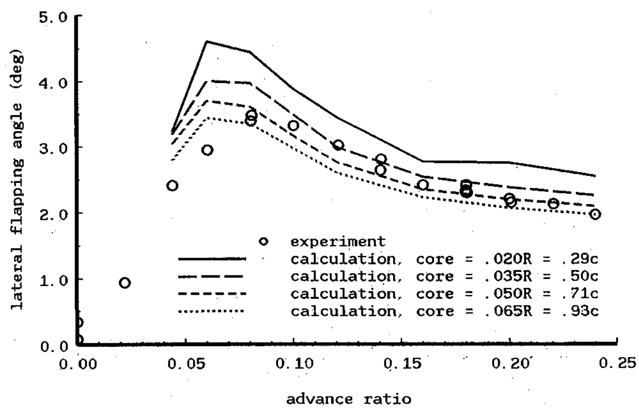

Fig. 1 Influence of core size on calculated lateral flapping of a model rotor: first-order lifting-line theory, no lifting-surface theory correction.

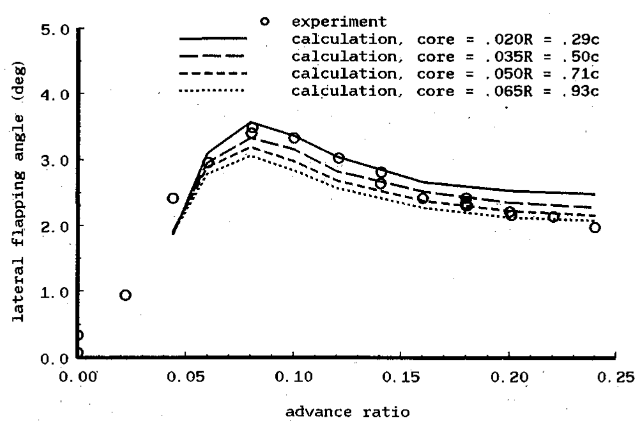

Fig. 2 Influence of core size on calculated lateral flapping of a model rotor: second-order lifting-line theory.

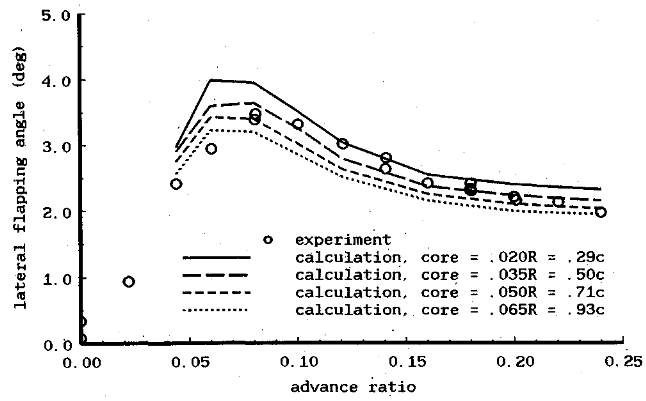

Fig. 3 Influence of core size on calculated lateral flapping of a model rotor: lifting-surface theory correction.

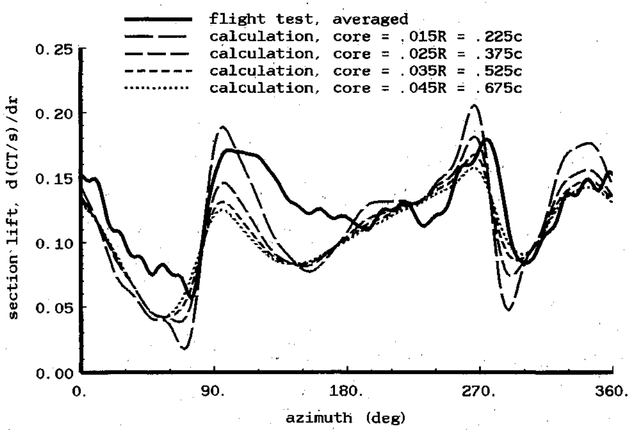

Fig. 4 Influence of core size on calculated 97% R loading of SA349/2 helicopter rotor: first-order lifting-line theory, no lifting-surface theory correction.

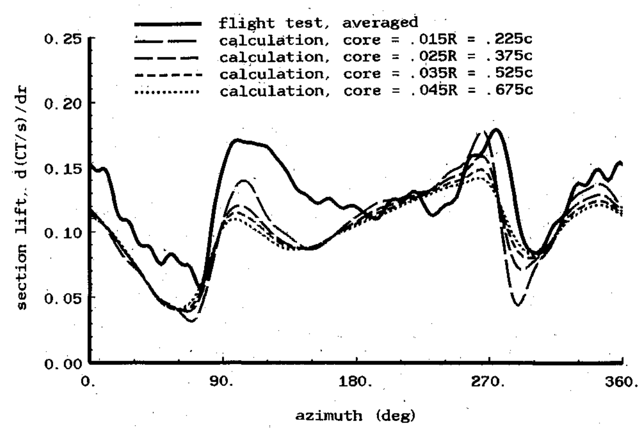

Fig. 5 Influence of core size on calculated 97% R loading of SA349/2 helicopter rotor: second-order lifting-line theory.

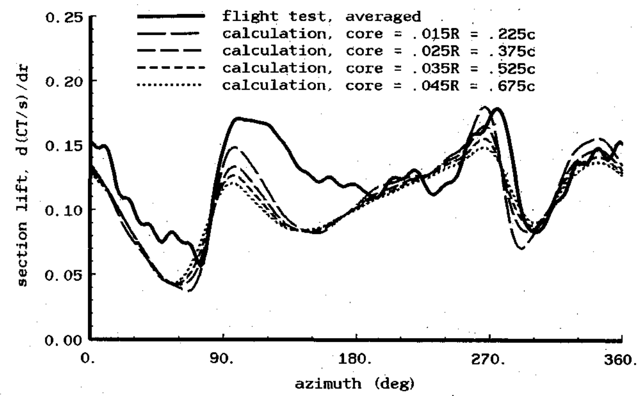

Fig. 6 Influence of core size on calculated 97% R loading of SA349/2 helicopter rotor: with lifting-surface theory correction.

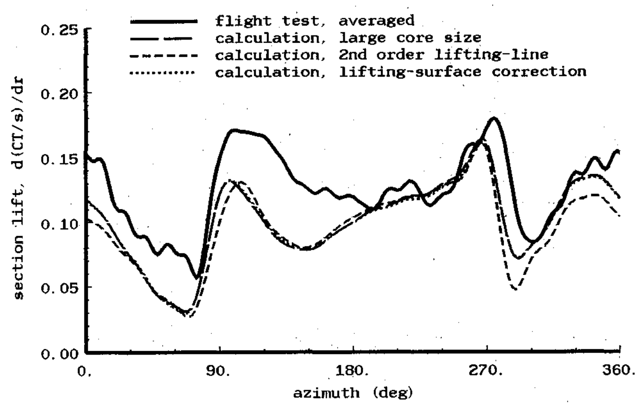

Fig. 7 Influence of blade-vortex interaction calculation on 97% R loading of SA349/2 helicopter rotor.

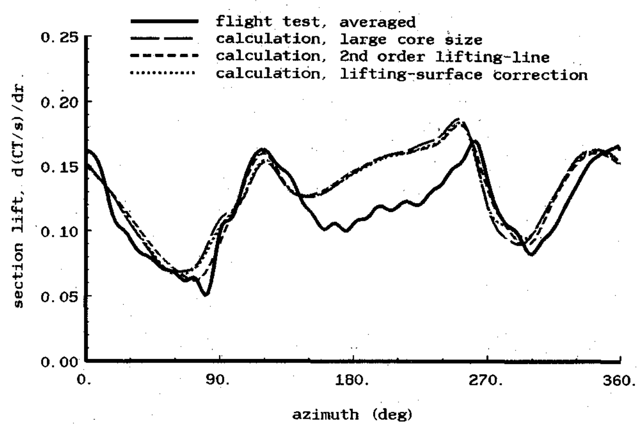

Fig. 8 Influence of blade-vortex interaction calculation on 88% R loading of SA349/2 helicopter rotor.

(50% chord) appears best. These two approaches produce about the same results, but with differences in details. The lifting-surface theory correction is nearly equivalent to simply increasing the core size by 0.015 R (20% chord). For advance ratios below about 0.06, the lifting-surface correction does not improve the correlation, whereas the second-order lifting-line theory gives good results over the entire speed range shown. In general, therefore, the best model is the second-order lifting-line theory. The core size appropriate with this model is still relatively large, which might be a Reynolds number effect on the actual viscous core. To improve the correlation for very low speeds, a better wake geometry calculation is needed.

SA349/2 Helicopter Flight Test

In the flight tests of an SA349/2 helicopter,[12] the low-speed cases exhibit significant blade-vortex interaction. Flight test case 2 is considered here (condition 1 of Ref. 5). The lift coefficient was measured at three radial stations and averaged over six revolutions. The measured lift shows considerable variation from revolution to revolution for this flight condition. However, an examination of the blade-vortex interaction peaks in the individual revolutions showed that the amplitude of the peaks in the averaged data were only 3–5% less than the mean of the amplitude of the peak in the original data; and the peak amplitudes in the original data vary by ±7–16% from

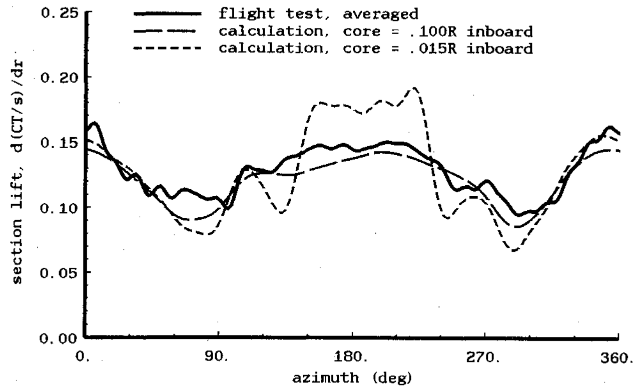

Fig. 9 Influence of inboard blade-vortex interaction on 75% R loading of SA349/2 helicopter rotor (with lifting-surface theory correction).

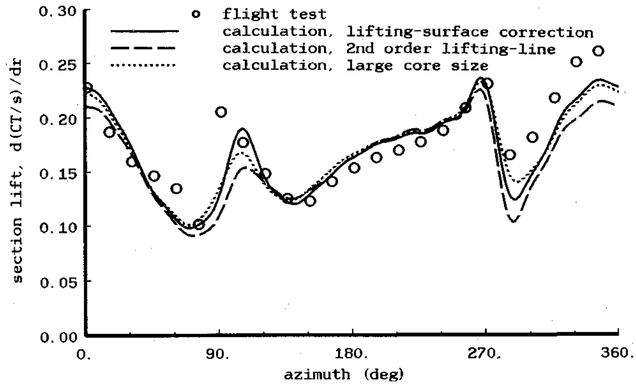

Fig. 10 Influence of blade-vortex interaction calculation on 95% R loading of H-34 helicopter rotor.

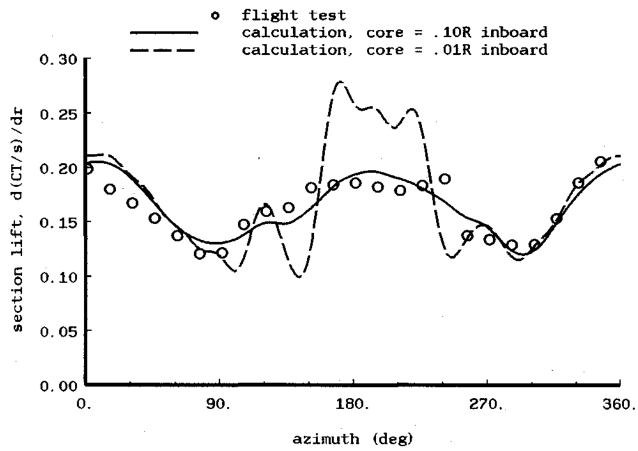

Fig. 11 Influence of inboard blade-vortex interaction on 75% R loading of H-34 helicopter rotor (with lifting-surface theory correction).

the mean amplitudes. Hence, it is acceptable to base the correlation with analysis on the averaged lift data. The rotor parameters and operating condition are summarized in Table 1.

It is known that the free-wake geometry must be used in order to accurately calculate the blade-vortex interaction loads for this low-speed flight condition.[5] Figures 4-6 show the influence of core size on the calculated loading at 97% R for the three blade-vortex interaction models (1-3 defined in the previous section). For the loading at the tip, the best core size is $0.025R$ (37.5% chord) for case 1, or $0.015R$ (22.5% chord) with either the second-order lifting-line theory or the lifting-surface correction. Figures 7 and 8 show the loading calculated at 97 and 88% R, using the best core size for each of the three models. All three models give similar results, although the details of the calculated loading vary. Note that the lifting-surface theory correction is nearly equivalent to simply increasing the core radius by $0.01R$ (15% chord).

With either second-order lifting-line theory or the lifting-surface theory correction, the required core size of 22.5% chord is in the expected range for the physical viscous core radius. The correlation suggests that the actual core radius varies with azimuth, being larger for vortices created on the retreating side of the disk. Although the amplitudes of the blade-vortex interaction peaks are well matched, the calculated load is low in the second quadrant for 97% R, and high in the third quadrant for 88% R.

It has been observed[10] that when the vortex-induced loads are calculated using a core size that gives good correlation at the blade tip, the strength of the blade-vortex interactions is significantly overpredicted for inboard radial stations. Figure 9 illustrates the effect for the SA349/2 data (recall that a core size of $0.015R$ gave good results for the tip). The core size can be increased for collocation points on the inboard part of the blade in order to eliminate (but not explain) this problem. Figure 9 shows the improved correlation using a core size of $0.100R$ for radial stations inboard of 80% R (transitioning to $0.015R$ at 90% R). Evidently there is some phenomenon limiting the loads. Several possibilities have been proposed: local distortion of the vortex geometry; bursting of the vortex core, induced by the blade; vortex interaction with the trailed wake it induces behind the blade, with the effect of diffusing the circulation in the vortex; and local flow separation produced by the high radial pressure gradients on the blade. The exact physical mechanism involved remains speculative, however. Increasing the core size for inboard collocation points is a simple way to model the effect on airloads, but no explanation of the physics is intended. More detailed measurements of the aerodynamics, including the wake geometry, are needed to explore this phenomenon.

If the lifting-surface theory correction is applied to every tip vortex line segment in the wake model, the time required to compute the wake influence coefficients is doubled. In order to reduce the computation time, the correction can be applied for a vortex line segment only if the distance between the collocation point and the center of the segment is less than a distance d_{ls}. For this case, it was found that the calculated loads were unchanged for d_{ls}/R varying from 100. to 0.5, and were nearly the same for $d_{ls}/R = 0.2$; however, with $d_{ls}/R = 0.1$, the loads were often the same as without the correction. Hence, it is concluded that it is possible to apply the lifting-surface correction with $d_{ls}/R = 0.5$, which increases the influence coefficient computation time by only 5%.

H-43 Helicopter Flight Test

The flight tests of an H-34 helicopter[13] have long been a standard for rotor airloads data. Calculations for the H-34 rotor produce results similar to the SA349/2, demonstrating the consistency of the present theory. In the H-34 flight tests, the section lift was measured at seven radial stations, and the data from oscillograph records was averaged over three revolutions. The rotor parameters and operating conditions are summarized in Table 1.

Figure 10 shows the loading calculated for 95% R using the three blade-vortex interaction models, with the best core size for each. All three give generally similar results, but with differing details. With the lifting-surface correction or the second-order lifting-line theory, a core size of about 21% chord is good, just as for the SA349/2 calculation. Again, a larger core size on the retreating side and a smaller core size on the advancing side would improve the correlation. The lifting-surface correction is roughly equivalent to increasing the core size by 15% chord, as for the SA349/2. Figure 11 shows the effect of the core size for inboard collocation points on the calculated loads. Suppressing the blade-vortex interaction inboard is essential for good correlation.

Swept Tip and Yawed Flow

Second-order lifting-line theory should also improve the calculation of the blade loading on swept tips and in yawed flow. On a helicopter rotor in high-speed forward flight, the blades encounter large yaw angles over most of the disk. Calculations for typical cases showed a significant difference between first-order and second-order lifting-line theories. Unfortunately, no measured airloads data were available for a rotor with a swept tip, and at high speeds other factors (such as blade motion) can have as much effect. Hence, the calculations performed for swept tips and high speed served primarily to insure that the second-order lifting-line theory was not exhibiting anomalous effects.

Some aspects of the theory are worth noting. First, second-order lifting-line theory for a swept tip involves both the position of collocation points, and sweep of the lifting-line (bound vortex). Using a straight lifting line for a swept blade is an approximation, but might be useful. Using a quarter-chord collocation point with a swept lifting line is not consistent, and should not be considered. Second, sweep and yawed flow must be considered in both the inner problem and the outer (wake) problem. For example, sweep of the tip introduces angle of attack and Mach number corrections that significantly affect the loads. Third, although second-order lifting-line theory and the lifting-surface theory correction are alternative methods for improving the calculation of blade-vortex interaction loads, the lifting-surface theory correction has no role in the aerodynamics of a swept tip or yawed flow.

Conclusions

Two alternative approaches for blade-vortex interaction were considered: second-order lifting-line theory, and a lifting-surface theory correction. Good correlation with measured rotor data was shown for cases involving significant blade-vortex interaction. For full-scale data, the vortex core size required for the correlation was about 20% chord. This is in the range of measured core sizes for rotors, but is still probably too large. The core size must account for not only the physical viscous core radius, but also all those aspects of the blade-vortex interaction that are not otherwise included in the analysis. Factors that would cause the core size in the analysis to be still too large include:

1) Discretization of the wake. Generally, the radial and azimuthal resolution in the discretized wake is too large, producing an overprediction of vortex-induced loads.

2) Partial tip vortex rollup. Contrary to the assumptions of the analysis, the tip vortex may not be completely rolled up by the time it reaches the following blade, i.e., the strength may be less than the value of the peak bound circulation.

3) Unsteadiness and noise in the data. Particularly if the azimuth angle of the blade-vortex interactions changes from rev to rev, the averaging process will reduce the measured peak loads.

A factor that would cause the core size to be too small is discretization of the calculated lift: the blade section loading is calculated here with an azimuth resolution of 15 deg, which would generally have the effect of reducing the apparent peak loads.

The common approach of using a larger vortex core radius to account for lifting-surface effects was quantified: the core radius should be increased by about 15% chord if neither second-order lifting-line theory nor the lifting-surface correction are used. The second-order lifting-line theory also improves the modeling of low-aspect ratio blades, yawed flow, and swept tips. Although the analysis is apparently functioning properly, test data were not available to verify the treatment of yawed flow and swept tips.

Discrepancies between analysis and test data suggest that the distorted wake geometry, tip vortex formation, and wake rollup must be investigated further. This conclusion is a common result of any airloads correlation work. Information is needed about the self-induced distortion of the entire wake, the tip vortex core sizes and strength, and the structure of the inboard wake. Questions and observations from the present investigation include the following.

1) The correlation suggests that the core size is larger for vortices formed on the retreating side than for those formed on the advancing side.

2) Something is happening on the inboard part of the blade to reduce the measured vortex-induced loads.

3) The free-wake analysis used does not calculate the distortion of the inboard wake.

4) Examination of the spanwise circulation distributions, particularly with highly twisted blades, suggests that the tip vortices may not be strongly rolled up on the advancing side.

Both theoretical and experimental investigations are recommended. Many phenomena involved in rotor wakes will remain speculative until measurements are available for at least the positions of the tip vortices, and the structure and extent of the wake rollup. Wake measurements should be made simultaneously with airloads measurements, and at full-scale Reynolds numbers. The theoretical work need not wait for the test data, however, since there are clear deficiencies in the existing wake geometry and rollup calculations.

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